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Relationships between axisymmetric bending and buckling solutions of FGM circular plates based on third-order plate theory and classical plate theory

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Abstract

The third-order shear deformation plate theory (TPT) is employed to solve the axisymmetric bending and buckling problems of functionally graded circular plates. Relationships between the TPT solutions of axisymmetric bending and buckling of functionally graded circular plates and those of isotropic circular plates based on the classical plate theory (CPT) are presented, from which one can easily obtain the TPT solutions for the axisymmetric bending and buckling of functionally graded plates. It is assumed in analysis that the mechanical properties of the functionally graded plates vary continuously through the thickness of the plate and obey a power law distribution of the volume fraction of the constituents. Effects of material gradient property and shear deformation on the bending and buckling of functionally graded plates are discussed in the frameworks of the first-order plate theory (FPT) and third-order plate theories. Also, comparisons of the TPT solutions to the FPT and CPT solutions are presented, which show that the first-order shear deformation plate theory is enough to consider the effect of shear deformation on the axisymmetric bending and buckling of functionally graded circular plate, a much higher order and more complex plate theory (say TPT) is not necessary for such a kind of problem.

Keywords: Functionally graded material; FGM; Circular plate; Bending; Buckling; Third-order plate theory; The first-order plate theory; Classical plate theory

1. Introduction

There are few works on the bending and buckling of functionally graded structures in contrary to the extensive investigations on isotropic and composite plates and shells. Using finite element method, Praveen and Reddy (1998) studied the static and dynamic responses of functionally graded ceramic-metal plate

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accounting for the transverse shear deformation, rotary inertia and moderately large rotations in the von Karman sense, in which effect of imposed temperature field on the response of the functionally graded plate was discussed in detail. Reddy and Chin (1998) investigated the dynamic thermo-elastic response of functionally graded cylinders and plates. A thermo-elastic boundary value problem was derived by using the first-order shear deformation plate theory (FPT) accounting for the coupling with three-dimensional heat conduction equation for a functionally graded plate. Based on the higher-order shear deformation theory of plate, both theoretical and finite element formulations for thick FGM plates were developed by Reddy (2000), and the nonlinear dynamic response of FGM plates subjected to suddenly applied uniform pressure were studied. Woo and Meguid (2001) derived an analytical solution expressed in terms of Fourier series for the large deflection of functionally graded plates and shallow shells under transverse mechanical loading and a temperature field in the framework of von Karman plate theory. Assuming that the material properties throughout the structure are produced by a spatial distribution of the local reinforcement volume fraction $v_f = v_f(x, y, z)$, Feldman and Aboudi (1997) studied the elastic bifurcation buckling of functionally graded plate under in-plane compressive loading. Thermal buckling of functionally graded rectangular thin plate were studied by Javaheri and Eslami (2002a,b), governing equations of the plate were derived based on the classical and the higher-order shear deformation theories of plate, respectively, and closed form solutions were obtained under several types of thermal loads. More recently, nonlinear bending and post-buckling of functionally graded circular plates subjected to mechanical and thermal loadings were studied by Ma and Wang (2003a,b).

Owing to the mathematical similarity of the governing equations of bending and buckling problems between the classical and the third-order plate theories, it enable one to derive the solutions based on the higher-order plate theory in terms of those based on the classical plate theory. Extensive works on the relationships between the solutions of beams and plates based on the higher-order plate theory and those based on the classical solutions were carried out (Reddy and Wang, 1997, 1998, 2000; Wang and Lee, 1998; Wang and Reddy, 1997) and summarized by Wang et al. (2000). It should be noted that most of the aforementioned works focused on the isotropic and composite beams and plates. For functionally graded plates, Reddy et al. (1999) obtained the FPT solutions of the axisymmetric bending of functionally graded Mindlin annular and circular plates, in which the solutions are expressed in terms of the solutions based on the classical plate theory (CPT). See Wang et al. (2000) for more details.

In the present work, we will employ the third-order shear deformation plate theory to solve the bending and buckling problems of functionally graded circular plates. The aforementioned method are extended to derive the relationships between the solutions of the axisymmetric bending and buckling of functionally graded circular plates based on the third-order shear deformation plate theory (called TPT solutions) and those of isotropic circular plates based on the classical plate theory (called CPT solutions). Such that one can easily obtain the TPT solutions for the axisymmetric bending and buckling of FGM circular plates expressed in terms of the well-known CPT solutions for isotropic circular plates. The relationships may be used to check the validity, convergence and accuracy of numerical results for FGM plate analysis. Moreover, Effects of the gradient of material property and shear deformation on the axisymmetric bending and buckling of functionally graded plates are discussed in the frameworks of FPT and TPT, and comparisons of the TPT solutions to the FPT and CPT solutions are presented.

2. Axisymmetric bending problem

A solid functionally graded circular plate with radius b and thickness h is considered here. The cylindrical coordinates r, θ and z are used in analysis. r axis is taken radially outward from the center of the plate, θ axis is taken along a circumference of the plate, r–θ plane is taken to be the undeformed mid-plane of the plate, and z axis is perpendicular to the r–θ plane.

Generally, Poisson's ratio υ varies in a small range. For simplicity, we assume υ be a constant for functionally graded materials. Moreover, we assume Young's modulus E varies along the thickness of the plate and obey the following relation (Praveen and Reddy, 1998; Reddy and Chin, 1998)

$$E(z) = (E_{\mathrm{m}} - E_{\mathrm{c}})\left(\frac{h-2z}{2h}\right)^n + E_{\mathrm{c}}, \tag{1}$$

where the subscripts m and c denote the metallic and ceramic constituents, respectively, and n is a material constant. Thus, constitutive relations of the functionally graded materials can be expressed as

$$\sigma_r = \frac{E(z)}{1-\upsilon^2}(\varepsilon_r + \upsilon\varepsilon_\theta), \tag{2a}$$

$$\sigma_\theta = \frac{E(z)}{1-\upsilon^2}(\upsilon\varepsilon_r + \varepsilon_\theta), \tag{2b}$$

$$\tau_{rz} = \frac{E(z)}{2(1+\upsilon)}\gamma_{rz}. \tag{2c}$$

2.1. Basic equations based on the third-order shear deformation plate theory (TPT)

The third-order shear deformation plate theory is based on the following displacement field (Reddy, 1984)

$$U_r(r,z) = u(r) + z\phi(r) - \alpha z^3\left(\phi + \frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{3a}$$

$$U_z(r,z) = w(r), \tag{3b}$$

where U_r and U_z are the displacements along the coordinates r and z, respectively, u and w are the displacements in the mid-plane of the plate along the coordinates r and z, respectively, ϕ denotes the slope at $z = 0$ of the deformed line that was straight in the undeformed plate, and $\alpha = 4/3h^2$. Using the following relations (Timshenko and Woinowsky-Krieger, 1959)

$$\varepsilon_r = \frac{\partial U_r}{\partial r},$$

$$\varepsilon_\theta = \frac{U_r}{r},$$

$$\gamma_{rz} = \frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r},$$

one then obtains

$$\varepsilon_r = \frac{\mathrm{d}u}{\mathrm{d}r} + z\frac{\mathrm{d}\phi}{\mathrm{d}r} - \alpha z^3\left(\frac{\mathrm{d}\phi}{\mathrm{d}r} + \frac{\mathrm{d}^2 w}{\mathrm{d}r^2}\right), \tag{4a}$$

$$\varepsilon_\theta = \frac{u}{r} + z\frac{\phi}{r} - \alpha z^3\left(\frac{\phi}{r} + \frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{4b}$$

$$\gamma_{rz} = \phi + \frac{\mathrm{d}w}{\mathrm{d}r} - 3\alpha z^2\left(\phi + \frac{\mathrm{d}w}{\mathrm{d}r}\right). \tag{4c}$$

Using the principle of virtual displacements, one can derive the following equilibrium equations for the Reddy plate,

$$\frac{\mathrm{d}(rN_r)}{\mathrm{d}r} - N_\theta = 0, \tag{5a}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\overline{M}_r) - \overline{M}_\theta - r\overline{Q}_r = 0, \tag{5b}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\overline{Q}_r) + \alpha\frac{\mathrm{d}^2}{\mathrm{d}r^2}(rP_r) - \alpha\frac{\mathrm{d}P_\theta}{\mathrm{d}r} + rq = 0, \tag{5c}$$

where

$$(N_r, M_r, P_r) = \int_{-h/2}^{h/2} \sigma_r(1, z, z^3)\,\mathrm{d}z, \tag{5d}$$

$$(N_\theta, M_\theta, P_\theta) = \int_{-h/2}^{h/2} \sigma_\theta(1, z, z^3)\,\mathrm{d}z, \tag{5e}$$

$$(Q_r, R_r) = \int_{-h/2}^{h/2} \tau_{rz}(1, z^2)\,\mathrm{d}z, \tag{5f}$$

$$\overline{M}_r = M_r - \alpha P_r, \tag{5g}$$

$$\overline{M}_\theta = M_\theta - \alpha P_\theta, \tag{5h}$$

$$\overline{Q}_r = Q_r - 3\alpha R_r \tag{5i}$$

with q is the transverse load.

The stress resultants and displacement relations can be expressed as

$$N_r = A_{11}\left(\frac{\mathrm{d}u}{\mathrm{d}r} + \upsilon\frac{u}{r}\right) + \overline{B}_{11}\left(\frac{\mathrm{d}\phi}{\mathrm{d}r} + \upsilon\frac{\phi}{r}\right) - \alpha E_{11}\left(\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{\upsilon}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{6a}$$

$$N_\theta = A_{11}\left(\upsilon\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{u}{r}\right) + \overline{B}_{11}\left(\upsilon\frac{\mathrm{d}\phi}{\mathrm{d}r} + \frac{\phi}{r}\right) - \alpha E_{11}\left(\upsilon\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{6b}$$

$$M_r = B_{11}\left(\frac{\mathrm{d}u}{\mathrm{d}r} + \upsilon\frac{u}{r}\right) + \overline{D}_{11}\left(\frac{\mathrm{d}\phi}{\mathrm{d}r} + \upsilon\frac{\phi}{r}\right) - \alpha F_{11}\left(\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{\upsilon}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{7a}$$

$$M_\theta = B_{11}\left(\upsilon\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{u}{r}\right) + \overline{D}_{11}\left(\upsilon\frac{\mathrm{d}\phi}{\mathrm{d}r} + \frac{\phi}{r}\right) - \alpha F_{11}\left(\upsilon\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{7b}$$

$$Q_r = \overline{A}_{44}\left(\phi + \frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{8}$$

$$P_r = E_{11}\left(\frac{\mathrm{d}u}{\mathrm{d}r} + \upsilon\frac{u}{r}\right) + \overline{F}_{11}\left(\frac{\mathrm{d}\phi}{\mathrm{d}r} + \upsilon\frac{\phi}{r}\right) - \alpha H_{11}\left(\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{\upsilon}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{9a}$$

$$P_\theta = E_{11}\left(\upsilon\frac{\mathrm{d}u}{\mathrm{d}r}+\frac{u}{r}\right)+\overline{F}_{11}\left(\upsilon\frac{\mathrm{d}\phi}{\mathrm{d}r}+\frac{\phi}{r}\right)-\alpha H_{11}\left(\upsilon\frac{\mathrm{d}^2w}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{9b}$$

$$R_r = \overline{D}_{44}\left(\phi+\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{10}$$

where

$$(A_{11},B_{11},D_{11},E_{11},F_{11},H_{11}) = \int_{-h/2}^{h/2}\frac{E(z)}{1-\upsilon^2}\left(1,z,z^2,z^3,z^4,z^6\right)\mathrm{d}z,$$

$$(A_{44},D_{44},F_{44}) = \int_{-h/2}^{h/2}\frac{E(z)}{2(1+\upsilon)}(1,z^2,z^4)\,\mathrm{d}z,$$

$$\overline{B}_{11} = B_{11}-\alpha E_{11},$$

$$\overline{D}_{11} = D_{11}-\alpha F_{11},$$

$$\overline{F}_{11} = F_{11}-\alpha H_{11},$$

$$\overline{A}_{44} = A_{44}-3\alpha D_{44},$$

$$\overline{D}_{44} = D_{44}-3\alpha F_{44},$$

$$\widehat{D}_{11} = \overline{D}_{11}-\alpha\overline{F}_{11}.$$

2.2. Basic equations based on the classical plate theory (CPT)

The classical plate theory is based on the following displacement field (Reddy et al., 1999)

$$U_r(r,z) = u(r) - z\frac{\mathrm{d}w}{\mathrm{d}r}, \tag{11a}$$

$$U_z(r,z) = w(r). \tag{11b}$$

Equilibrium equations and the stress resultants and displacement relations are given by

$$\frac{\mathrm{d}(rN_r^{\mathrm{C}})}{\mathrm{d}r} - N_\theta^{\mathrm{C}} = 0, \tag{12a}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}(rM_r^{\mathrm{C}}) - M_\theta^{\mathrm{C}} - rQ_r^{\mathrm{C}} = 0, \tag{12b}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}(rQ_r^{\mathrm{C}}) + rq = 0, \tag{12c}$$

$$M_r^{\mathrm{C}} = -D\left(\frac{\mathrm{d}^2w^{\mathrm{C}}}{\mathrm{d}r^2}+\upsilon\frac{1}{r}\frac{\mathrm{d}w^{\mathrm{C}}}{\mathrm{d}r}\right), \tag{13a}$$

$$M_\theta^{\mathrm{C}} = -D\left(\upsilon\frac{\mathrm{d}^2w^{\mathrm{C}}}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}w^{\mathrm{C}}}{\mathrm{d}r}\right), \tag{13b}$$

where

$$D = \frac{h}{12(1-\upsilon^2)}E$$

and the quantities with superscript C refer to the classical plate theory.

2.3. Bending solutions

From Eqs. (5a) and (6), one obtains

$$u = \alpha\frac{E_{11}}{A_{11}}\frac{\mathrm{d}w}{\mathrm{d}r} - \frac{\overline{B}_{11}}{A_{11}}\phi + C_1 r + C_2\frac{1}{r}, \tag{14}$$

where C_1 and C_2 are integral constants. Substituting Eq. (14) into Eqs. (7) and (9), one obtains

$$M_r = \widehat{\Omega}_1\left(\frac{\mathrm{d}\phi}{\mathrm{d}r} + \upsilon\frac{\phi}{r}\right) - \alpha\Omega_2\left(\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \upsilon\frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right) + B_{11}(1+\upsilon)C_1 - \frac{1}{r^2}B_{11}(1-\upsilon)C_2, \tag{15a}$$

$$M_\theta = \widehat{\Omega}_1\left(\upsilon\frac{\mathrm{d}\phi}{\mathrm{d}r} + \frac{\phi}{r}\right) - \alpha\Omega_2\left(\upsilon\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right) + B_{11}(1+\upsilon)C_1 + \frac{1}{r^2}B_{11}(1-\upsilon)C_2, \tag{15b}$$

$$P_r = \widehat{\Omega}_2\left(\frac{\mathrm{d}\phi}{\mathrm{d}r} + \upsilon\frac{\phi}{r}\right) - \alpha\Omega_3\left(\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \upsilon\frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right) + E_{11}(1+\upsilon)C_1 - \frac{1}{r^2}E_{11}(1-\upsilon)C_2, \tag{16a}$$

$$P_\theta = \widehat{\Omega}_2\left(\upsilon\frac{\mathrm{d}\phi}{\mathrm{d}r} + \frac{\phi}{r}\right) - \alpha\Omega_3\left(\upsilon\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right) + E_{11}(1+\upsilon)C_1 + \frac{1}{r^2}E_{11}(1-\upsilon)C_2, \tag{16b}$$

where

$$\Omega_1 = D_{11} - B_{11}^2/A_{11},$$

$$\Omega_2 = F_{11} - B_{11}E_{11}/A_{11},$$

$$\Omega_3 = H_{11} - E_{11}^2/A_{11},$$

$$\widehat{\Omega}_1 = \Omega_1 - \alpha\Omega_2,$$

$$\widehat{\Omega}_2 = \Omega_2 - \alpha\Omega_3.$$

In what follows, we will use the following moment sum M^{C} and M for classical and the third-order plate theory, respectively, and higher-order moment sums P for the third-order plate theory (Reddy and Wang, 1997),

$$M^{\mathrm{C}} = \frac{M_r^{\mathrm{C}} + M_\theta^{\mathrm{C}}}{1+\upsilon}, \tag{17a}$$

$$M = \frac{M_r + M_\theta}{1+\upsilon}, \tag{17b}$$

$$P = \frac{P_r + P_\theta}{1+\upsilon}. \tag{17c}$$

From Eqs. (13) and (12b), one obtains

$$M^{\mathrm{C}} = -D\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}w^{\mathrm{C}}}{\mathrm{d}r}\right) \tag{18}$$

and

$$r\frac{\mathrm{d}M^{\mathrm{C}}}{\mathrm{d}r} = rQ_r^{\mathrm{C}}. \tag{19}$$

Similarly, we have from Eqs. (15) and (16) that

$$M = \widehat{\Omega}_1\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi) - \alpha\Omega_2\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}w}{\mathrm{d}r}\right) + B_{11}C_1, \tag{20}$$

$$P = \widehat{\Omega}_2\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi) - \alpha\Omega_3\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}w}{\mathrm{d}r}\right) + E_{11}C_1, \tag{21}$$

$$r\frac{\mathrm{d}M}{\mathrm{d}r} = \frac{\mathrm{d}}{\mathrm{d}r}(rM_r) - M_\theta, \tag{22}$$

$$r\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{\mathrm{d}}{\mathrm{d}r}(rP_r) - P_\theta. \tag{23}$$

Letting the effective shear force in the third-order plate theory being

$$rV_r \equiv r\overline{Q}_r + \alpha\left[\frac{\mathrm{d}}{\mathrm{d}r}(rP_r) - P_\theta\right], \tag{24}$$

one then obtains from Eq. (5b) that

$$rV_r = r\frac{\mathrm{d}M}{\mathrm{d}r}. \tag{25}$$

Moreover, we have from Eqs. (5b), (5c) and (22)

$$\frac{\mathrm{d}}{\mathrm{d}r}(rV_r) + rq = 0. \tag{26}$$

From Eqs. (12c), (26) and (25), we obtain

$$rV_r = rQ_r^{\mathrm{C}} + C_3, \tag{27a}$$

$$r\frac{\mathrm{d}M}{\mathrm{d}r} = rQ_r^{\mathrm{C}} + C_3. \tag{27b}$$

From Eqs. (27b) and (19), we have

$$M = M^{\mathrm{C}} + C_3\ln r + C_4, \tag{28}$$

where C_3 and C_4 are integral constants. Substituting Eq. (18) into Eq. (28), we have

$$M = -D\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}w^{\mathrm{C}}}{\mathrm{d}r}\right) + C_3\ln r + C_4. \tag{29}$$

From Eqs. (20) and (29), one obtains

$$\phi-\alpha\frac{\Omega_2}{\widehat{\Omega}_1}\frac{\mathrm{d}w}{\mathrm{d}r}+\frac{B_{11}}{2\widehat{\Omega}_1}C_1r=-\frac{D}{\widehat{\Omega}_1}\frac{\mathrm{d}w^{\mathrm{C}}}{\mathrm{d}r}+\frac{C_3}{4\widehat{\Omega}_1}r(2\ln r-1)+\frac{C_4}{2\widehat{\Omega}_1}r+\frac{C_5}{\widehat{\Omega}_1}\frac{1}{r}, \tag{30}$$

where C_5 is a constant. Such that Eq. (8) can be rewritten as

$$r\phi=\frac{1}{\overline{A}_{44}}rQ_r-r\frac{\mathrm{d}w}{\mathrm{d}r}. \tag{31}$$

Substituting Eq. (31) into Eqs. (20) and (21), one obtains

$$M=\frac{\widehat{\Omega}_1}{\overline{A}_{44}}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rQ_r)-\Omega_1\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}w}{\mathrm{d}r}\right)+B_{11}C_1, \tag{32}$$

$$P=\frac{\widehat{\Omega}_2}{\overline{A}_{44}}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rQ_r)-\Omega_2\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}w}{\mathrm{d}r}\right)+E_{11}C_1. \tag{33}$$

From Eq. (32), we have

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}w}{\mathrm{d}r}\right)=\frac{\widehat{\Omega}_1}{\Omega_1\overline{A}_{44}}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rQ_r)-\frac{M}{\Omega_1}+\frac{B_{11}}{\Omega_1}C_1. \tag{34}$$

Substituting Eq. (34) into Eq. (33), one obtains

$$P=-\alpha\frac{\overline{\Omega}}{\Omega_1\overline{A}_{44}}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rQ_r)+\frac{\Omega_2}{\Omega_1}M+\frac{E_{11}\Omega_1-B_{11}\Omega_2}{\Omega_1}C_1, \tag{35}$$

where

$$\overline{\Omega}=\Omega_1\Omega_3-\Omega_2^2.$$

Using Eqs. (5i), (22) and (23), Eq. (5b) can be expressed as

$$r(Q_r-3\alpha R_r)=r\frac{\mathrm{d}M}{\mathrm{d}r}-\alpha r\frac{\mathrm{d}P}{\mathrm{d}r}. \tag{36}$$

From Eqs. (10) and (31), we have

$$R_r=\frac{\overline{D}_{44}}{\overline{A}_{44}}Q_r.$$

Substituting R_r and Eqs. (27b) and (35) into Eq. (36), we obtain

$$\alpha^2\frac{\overline{\Omega}}{\Omega_1\overline{A}_{44}}r\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rQ_r)\right]-\frac{\widehat{A}_{44}}{\overline{A}_{44}}(rQ_r)=-\frac{\widehat{\Omega}_1}{\Omega_1}rQ_r^{\mathrm{C}}-C_3, \tag{37}$$

where

$$\widehat{A}_{44}=\overline{A}_{44}-3\alpha\overline{D}_{44}.$$

From Eq. (37) one can obtain the solution of shear force. In what follows, we will establish the relationships between deflection w of the FGM plate based on the third-order plate theory and deflection w^{C} of the isotropic plate based on the classical plate theory. Substituting Eq. (29) into Eq. (34) and integrating the resulted equation with respect to r, one then obtains the relation between w and w^{C}

$$w=\frac{D}{\Omega_1}w^{\mathrm{C}}+\frac{1}{4\Omega_1}\left[B_{11}C_1r^2-C_3r^2(\ln r-1)-C_4r^2-4C_5\ln r-4C_6\right]+\frac{\widehat{\Omega}_1}{\Omega_1\overline{A}_{44}}\int Q_r\,\mathrm{d}r, \tag{38}$$

where C_6 is a constant. Substituting Eq. (38) into Eq. (30), one obtains

$$\phi = -\frac{D}{\Omega_1}\frac{\mathrm{d}w^{\mathrm{C}}}{\mathrm{d}r} + \frac{\alpha\Omega_2}{\Omega_1\overline{A}_{44}}Q_r + \frac{1}{4\Omega_1}\left[-2B_{11}C_1r + C_3r(\ln r - 1) + 2C_4r + 4C_5\frac{1}{r}\right]. \tag{39}$$

Now, we consider a solid circular plate subjected to uniformly distributed load of intensity q. Using the continuous and symmetric conditions at the center of the plate, i.e. $r = 0$, one can easily see from Eq. (27a) that $C_3 = 0$. Such that Eq. (37) can be expressed as

$$\frac{\mathrm{d}^2Q_r}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}Q_r}{\mathrm{d}r} - \left(\frac{1}{r^2} + \beta^2\right)Q_r = \beta_0 Q_r^{\mathrm{C}}, \tag{40}$$

where

$$\beta^2 = \frac{\Omega_1\widehat{A}_{44}}{\alpha^2\overline{\Omega}},$$

$$\beta_0 = -\frac{\widehat{\Omega}_1\overline{A}_{44}}{\alpha^2\overline{\Omega}},$$

$$Q_r^{\mathrm{C}} = -\frac{1}{2}qr.$$

General solution of Eq. (40) can be expressed as

$$Q_r = A_1I_1(\beta r) + A_2K_1(\beta r) + C_0r, \tag{41}$$

where

$$C_0 = -\frac{q}{2}\frac{\widehat{\Omega}_1\overline{A}_{44}}{\Omega_1\widehat{A}_{44}},$$

with A_1 and A_2 being constants, and I_1 and K_1 being modified first-order Bessel functions of the first and second kinds, respectively.

If a solid circular plate is clamped at the upper and lower surfaces on the outer boundary but immovable in r direction, the continuous and symmetric conditions at the center of the plate, i.e. $r = 0$ give

$$A_2 = 0, \tag{42a}$$

$$C_2 = 0, \tag{42b}$$

$$C_5 = 0. \tag{42c}$$

The boundary conditions at $r = b$ give

$$C_1 = 0, \tag{43a}$$

$$C_4 = 0, \tag{43b}$$

$$A_1 = -\frac{C_0b}{I_1(\beta b)}, \tag{43c}$$

$$C_6 = \frac{\widehat{\Omega}_1C_0b^2}{2\overline{A}_{44}}\left[1 - \frac{2I_0(\beta b)}{\beta bI_1(\beta b)}\right]. \tag{43d}$$

Now, we obtain the TPT solution of deflection of the clamped FGM circular plate

$$w=\frac{D}{\Omega_1}w^{\mathrm{C}}+\frac{\widehat{\Omega}_1^2}{\Omega_1^2\widehat{A}_{44}}\frac{qb^2}{4}\left[1+\frac{I_0(\beta r)-I_0(\beta b)}{I_1(\beta b)}\frac{2}{\beta b}-\left(\frac{r}{b}\right)^2\right], \tag{44}$$

where the classical solution of the deflection w^{C} for the isotropic circular plates is

$$w^{\mathrm{C}}=\frac{qb^4}{64D}\left[1-\left(\frac{r}{b}\right)^2\right]^2.$$

Letting $\alpha=0$, one then obtains

$$\frac{\widehat{\Omega}_1^2}{\Omega_1^2\widehat{A}_{44}}=\frac{1}{A_{44}}$$

and

$$\frac{1}{\beta}=\alpha\left(\frac{\overline{\Omega}}{\Omega_1\widehat{A}_{44}}\right)^{1/2}=0.$$

Such that Eq. (44) reduces to the FPT solution of the FGM circular plate obtained by Reddy et al. (1999)

$$w=w^{\mathrm{F}}=\frac{D}{\Omega_1}w^{\mathrm{C}}+\frac{qb^2}{4A_{44}}\left[1-\left(\frac{r}{b}\right)^2\right], \tag{45}$$

where the transverse shear stiffness A_{44} should be changed to K_sA_{44} with K_s is the shear correction factor commonly taken to be 5/6.

For the isotropic circular plates, i.e. $n=0$ and $E(z)=E$, solution (44) reduces to the TPT solution for isotropic circular plate obtained by Reddy and Wang (1997)

$$w=w^{\mathrm{C}}+k_1\frac{qb^2}{4}\left[1+\frac{I_0(k_2r)-I_0(k_2b)}{I_1(k_2b)}\frac{2}{k_2b}-\left(\frac{r}{b}\right)^2\right], \tag{46}$$

where

$$k_1=\frac{6}{5Gh},$$

$$k_2=\frac{420(1-\upsilon)}{h^2},$$

and

$$G=\frac{E}{2(1+\upsilon)}.$$

To numerically compare the results obtained from the classical, the first-order and the third-order plate theories, a Aluminum/Zirconia functionally graded material is considered here. Young's moduli and Poisson's ratios are 70 GPa and 0.3 for Aluminum, and 151 GPa and 0.3 for Zirconia, respectively, which are taken from Praveen and Reddy (1998) and Reddy (2000). Fig. 1 shows variations of the maximum dimensionless deflection $w^*=64wD_c/(qb^4)$ of the clamped FGM circular plates with the ratio h/b, the plate thickness to the plate radius, for different values of material gradient constant n. The solid and dashed lines are the TPT and FPT results, respectively. It is seen that the maximum deflections of the FGM plates decrease with increasing the values of n, and increase with increasing the values of h/b due to the effect of transverse shear deformation. It should be noted that the TPT results are very close to the FPT results for

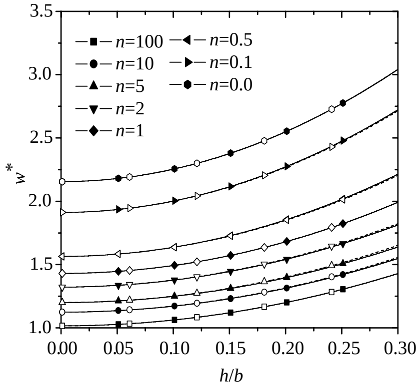

Fig. 1. The maximum dimensionless deflection of the clamped FGM plates vs h/b for different values of material gradient constant n. Solid lines: TPT solutions and dashed lines: FPT solutions.

the cases considered in the present paper. It could be concluded that the first-order shear deformation plate theory is enough to consider the effect of shear deformation on the axisymmetric bending of FGM circular plates.

In what follows, we will derive the TPT solutions for the axisymmetric buckling of FGM circular plates, and compare the TPT solutions with the CPT and FPT solutions.

3. Axisymmetric buckling problem

Considering a solid FGM circular plate subjected to uniformly distributed radial pressure p. Constitutive equations of the plate are given by Eqs. (2). Referencing to Javaheri and Eslami (2002b), one then obtains the following nonlinear geometric relations accounting for the moderately large deflection in the von Karman sense,

$$\varepsilon_r = \frac{\mathrm{d}u}{\mathrm{d}r} + \frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}r}\right)^2 + z\frac{\mathrm{d}\phi}{\mathrm{d}r} - \alpha z^3\left(\frac{\mathrm{d}\phi}{\mathrm{d}r} + \frac{\mathrm{d}^2 w}{\mathrm{d}r^2}\right), \tag{47a}$$

$$\varepsilon_\theta = \frac{u}{r} + z\frac{\phi}{r} - \alpha z^3\left(\frac{\phi}{r} + \frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right), \tag{47b}$$

$$\gamma_{rz} = \phi + \frac{\mathrm{d}w}{\mathrm{d}r} - 3\alpha z^2\left(\phi + \frac{\mathrm{d}w}{\mathrm{d}r}\right). \tag{47c}$$

To obtain the governing equations for the stability of the FGM plate, we assume

$$u = u_0 + u_1, \tag{48a}$$

$$\phi = \phi_0 + \phi_1, \tag{48b}$$

$$w = w_0 + w_1, \tag{48c}$$

where (u_0, ϕ_0, w_0) is a configuration on the primary path and its adjacent equilibrium configuration is (u, ϕ, w), (u_1, ϕ_1, w_1) are arbitrary small virtual increments. If one assumes (u_0, ϕ_0, w_0) being $(u_0, 0, 0)$, then the pre-buckling coupled mode is omitted.

According to the Trefftz criterion (Brush and Almroth, 1975), the governing equations for the buckling of the plate subjected to in-plane edge loading can be obtained by using the principle of virtual displacements

$$A_{11}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(ru_1)\right]+\overline{B}_{11}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1)\right]-\alpha E_{11}\frac{\mathrm{d}}{\mathrm{d}r}\nabla^2 w_1=0, \tag{49a}$$

$$\overline{B}_{11}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(ru_1)\right]+\widehat{D}_{11}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1)\right]-\alpha\overline{F}_{11}\frac{\mathrm{d}}{\mathrm{d}r}\nabla^2 w_1-\widehat{A}_{44}\left(\phi_1+\frac{\mathrm{d}w_1}{\mathrm{d}r}\right)=0, \tag{49b}$$

$$\alpha E_{11}\nabla^2\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(ru_1)\right]+\alpha\overline{F}_{11}\nabla^2\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1)\right]-\alpha^2 H_{11}\nabla^4 w_1+\widehat{A}_{44}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r\left(\phi_1+\frac{\mathrm{d}w_1}{\mathrm{d}r}\right)\right]$$
$$+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(rN_r^0\frac{\mathrm{d}w_1}{\mathrm{d}r}\right)=0, \tag{49c}$$

where

$$\nabla^2=\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r},$$

and N_r^0 corresponding to the $(u_0,0,0)$ is the pre-buckling radial force, i.e., $N_r^0=-p$.

The in-plane boundary condition is

$$A_{11}\left(\frac{\mathrm{d}u_1}{\mathrm{d}r}+\upsilon\frac{u_1}{r}\right)+\overline{B}_{11}\left(\frac{\mathrm{d}\phi_1}{\mathrm{d}r}+\upsilon\frac{\phi_1}{r}\right)-\alpha E_{11}\left(\frac{\mathrm{d}^2w_1}{\mathrm{d}r^2}+\upsilon\frac{1}{r}\frac{\mathrm{d}w_1}{\mathrm{d}r}\right)=0\quad \text{at } r=b; \tag{50}$$

for clamped boundary conditions,

$$\phi_1=0\quad \text{at } r=b, \tag{51a}$$

$$w_1=0\quad \text{at } r=b, \tag{51b}$$

$$\frac{\mathrm{d}w_1}{\mathrm{d}r}=0\quad \text{at } r=b. \tag{51c}$$

For simply supported boundary conditions,

$$w_1=0\quad \text{at } r=b, \tag{52a}$$

$$B_{11}\left(\frac{\mathrm{d}u_1}{\mathrm{d}r}+\upsilon\frac{u_1}{r}\right)+\overline{D}_{11}\left(\frac{\mathrm{d}\phi_1}{\mathrm{d}r}+\upsilon\frac{\phi_1}{r}\right)-\alpha F_{11}\left(\frac{\mathrm{d}^2w_1}{\mathrm{d}r^2}+\upsilon\frac{\mathrm{d}w_1}{\mathrm{d}r}\right)=0\quad \text{at } r=b, \tag{52b}$$

$$E_{11}\left(\frac{\mathrm{d}u_1}{\mathrm{d}r}+\upsilon\frac{u_1}{r}\right)+\overline{F}_{11}\left(\frac{\mathrm{d}\phi_1}{\mathrm{d}r}+\upsilon\frac{\phi_1}{r}\right)-\alpha H_{11}\left(\frac{\mathrm{d}^2w_1}{\mathrm{d}r^2}+\upsilon\frac{\mathrm{d}w_1}{\mathrm{d}r}\right)=0\quad \text{at } r=b. \tag{52c}$$

The continuous and symmetric conditions at the center of the plate ($r=0$)

$$u_1=0, \tag{53a}$$

$$\phi_1=0, \tag{53b}$$

$$\frac{\mathrm{d}w_1}{\mathrm{d}r}=0, \tag{53c}$$

$$V_r = 0. \tag{53d}$$

We know from the above mentioned continuous and symmetric conditions at $r = 0$ and boundary condition at $r = b$ that the integral constants $C_2 = 0$ and $C_1 = 0$ in Eq. (14). Such that, Eq. (14) can be rewritten as

$$A_{11}u_1 + \overline{B}_{11}\phi_1 = \alpha E_{11}\frac{\mathrm{d}w_1}{\mathrm{d}r} \tag{54}$$

and Eqs. (20) and (21) reduce to

$$M = \widehat{\Omega}_1\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1) - \alpha\Omega_2\nabla^2 w_1, \tag{55}$$

$$P = \widehat{\Omega}_2\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1) - \alpha\Omega_3\nabla^2 w_1. \tag{56}$$

We have from Eq. (55) that

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1) = \frac{M}{\widehat{\Omega}_1} + \alpha\frac{\Omega_2}{\widehat{\Omega}_1}\nabla^2 w_1. \tag{57}$$

From Eqs. (54) and (57), one obtains

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(ru_1) = \alpha\frac{E_{11}\Omega_1 - B_{11}\Omega_2}{A_{11}\widehat{\Omega}_1}\nabla^2 w_1 - \frac{\overline{B}_{11}M}{A_{11}\widehat{\Omega}_1}. \tag{58}$$

From Eq. (49b), one obtains

$$\widehat{A}_{44}r\left(\phi_1 + \frac{\mathrm{d}w_1}{\mathrm{d}r}\right) = \overline{B}_{11}r\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(ru_1)\right] + \widehat{D}_{11}r\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1)\right] - \alpha\overline{F}_{11}r\frac{\mathrm{d}}{\mathrm{d}r}\nabla^2 w_1. \tag{59}$$

Substituting Eqs. (59) and (58) into Eq. (49c), we have

$$\widehat{\Omega}_1\nabla^2\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1)\right] - \alpha\Omega_2\nabla^4 w_1 = p\nabla^2 w_1. \tag{60}$$

From Eq. (55), one obtains

$$\nabla^2 M = \widehat{\Omega}_1\nabla^2\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\phi_1)\right] - \alpha\Omega_2\nabla^4 w_1. \tag{61}$$

From Eqs. (60) and (61), one obtains

$$\nabla^2 M = p\nabla^2 w_1. \tag{62}$$

Substituting Eqs. (62), (57) and (58) into Eq. (49c), one obtains

$$\nabla^4 M - \xi\nabla^2 M - \zeta M = 0$$

which can be rewritten as

$$(\nabla^2 + \lambda_1)(\nabla^2 + \lambda_2)M = 0, \tag{63}$$

where

$$\lambda_{1,2} = -\xi/2 \pm \sqrt{(\xi/2)^2 + \zeta},$$

$$\xi = \zeta_1 + \zeta_2 p,$$

$$\zeta = \zeta_0 p,$$

$$\zeta_0 = \frac{\widehat{A}_{44}}{\alpha^2 \overline{\Omega}},$$

$$\zeta_1 = \frac{\widehat{A}_{44}\Omega_1}{\alpha^2 \overline{\Omega}},$$

$$\zeta_2 = -\frac{\widehat{\Omega}}{\alpha^2 \overline{\Omega}},$$

$$\widehat{\Omega} = \Omega_1 - 2\alpha\Omega_2 + \alpha^2\Omega_3.$$

For the plates with clamped and simply supported edge, we have

$$M = H_1 \quad \text{at } r = b, \tag{64a}$$

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 0 \quad \text{at } r = 0, \tag{64b}$$

where H_1 is a constant.

Since λ_2 in Eq. (63) is negative, it does not lead to a feasible buckling solution. Letting

$$m = (\nabla^2 + \lambda_2)M, \tag{65}$$

one then rewrites Eq. (63) as

$$(\nabla^2 + \lambda_1)m = 0. \tag{66}$$

Considering the fact that the moment sum M is a function of r only, we have

$$m = H_2 \quad \text{at } r = b, \tag{67}$$

where H_2 is a constant. Substituting Eqs. (57) and (58) into Eq. (59) and considering Eqs. (53b), (53c) and (64b), one obtains

$$\frac{\mathrm{d}}{\mathrm{d}r}\nabla^2 w_1 = 0 \quad \text{at } r = 0. \tag{68}$$

Noting Eqs. (62), (68) and (64b), we have

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 0 \quad \text{at } r = 0. \tag{69}$$

In what follows, we consider the axisymmetric buckling of isotropic Kirchhoff circular plates with clamped and simply supported edges. Using the Kirchhoff moment sum M^{C}, Wang (1996) derived the governing equations of the buckling of the plates under uniformly distributed radial pressure p

$$(\nabla^2 + \lambda^{\mathrm{C}})M^{\mathrm{C}} = 0 \tag{70}$$

with

$$M^{\mathrm{C}} = H_3, \quad \text{at } r = b, \tag{71a}$$

$$\frac{\mathrm{d}M^{\mathrm{C}}}{\mathrm{d}r} = 0, \quad \text{at } r = 0, \tag{71b}$$

where H_3 is a constant and

$$\lambda^{C} = \frac{p^{C}}{D} \tag{72}$$

with p^{C} being the critical buckling load of the isotropic plate based on the classical plate theory.

Based on the mathematic similarity between Eqs. (66), (67), (69) and Eqs. (70) and (71), one may deduce that

$$\lambda_1 = \lambda^{C}. \tag{73}$$

In view of Eq. (73), one can obtain the following relationship between the TPT solution of the critical buckling load p_F^R of FGM plate and the CPT solution of the critical buckling load p^C of isotropic plate

$$p_F^R = p^C \frac{\zeta_1 D + p^C}{\zeta_0 D^2 - \zeta_2 D p^C}. \tag{74}$$

If $\alpha = 0$, Eq. (74) reduces to the relationship between the FPT solution of critical buckling load p_F^F of FGM plate and the CPT solution of critical buckling load p^C of isotropic plate

$$p_F^F = p^C \frac{\Omega_1}{D + \frac{\Omega_1}{A_{44}} p^C}, \tag{75}$$

where

$$A_{44} = K_s \int_{-h/2}^{h/2} \frac{E(z)}{2(1+v)} \, dz.$$

Letting $n = 0$ and $E(z) = E$, the FGM plates become isotropic plates. One can easily obtain from Eqs. (74) and (75) that

$$p^R = p^C \frac{1 + \frac{p^C}{70Gh}}{1 + \frac{17p^C}{14Gh}}, \tag{76}$$

$$p^F = \frac{p^C}{1 + \frac{p^C}{K_s Gh}}. \tag{77}$$

Eqs. (76) and (77) are the relationships between the TPT and FTP solutions for the critical buckling loads p^R and p^F of isotropic plates and the CPT solution for the critical buckling load p^C of isotropic plate, which are identical with that obtained by Wang and Lee (1998).

The critical buckling load parameters pb^2/D for the clamped and the simply supported FGM circular plates mentioned in Section 2 are calculated based on the first-order and the third-order plate theories and plotted in Figs 2 and 3, respectively, for different values of h/b, ratio of the plate thickness to the plate radius. It is seen that values of critical buckling load parameters of the FGM plates increase with increasing the values of material gradient constant n, and decrease with increasing the values of h/b due to the effect of transverse shear deformation. It should be noted that the TPT solutions are almost the same as the FPT solutions for the cases considered in this paper. We could conclude that the first-order shear deformation plate theory is enough to consider the effect of shear deformation on the axisymmetric buckling of FGM circular plates.

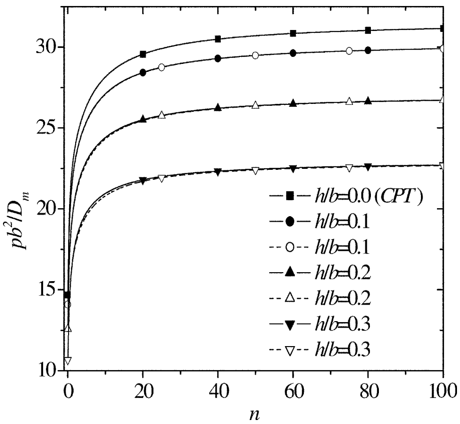

Fig. 2. Buckling load parameter of the clamped FGM plates vs material gradient constant n. Solid lines: TPT solutions and dashed lines: FPT solutions.

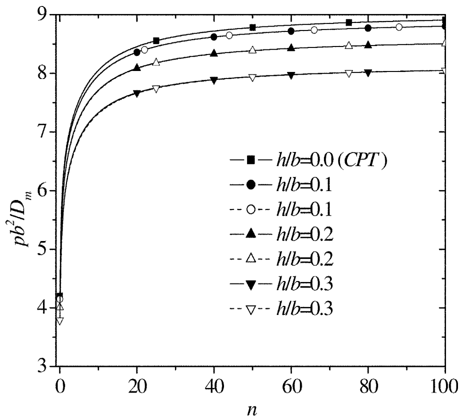

Fig. 3. Buckling load parameter of the simply supported FGM plates vs material gradient constant n. Solid lines: TPT solutions and dashed lines: FPT solutions.

4. Conclusions

The third-order shear deformation plate theory is employed to solve the axisymmetric bending and buckling problems of functionally graded circular plates. Based on the mathematical similarity of the governing equations of bending and buckling problems between the classical plate theory (CPT) and the third-order plate theory (TPT), relationships between the TPT solutions for the axisymmetric bending and buckling of FGM circular plates and the CPT solutions for isotropic plate have been derived, from which one can easily obtain the TPT solutions for the bending and buckling of functionally graded plates from the well-known CPT solutions. The relationships derived in the present paper could be used to check the validity, convergence and accuracy of numerical results for FGM plate analysis.

Effects of the material gradient property and shear deformation on the axisymmetric bending and buckling of FGM circular plates are discussed in the framework of the first-order plate theory (FPT) and TPT. Comparisons of the TPT solutions for bending and buckling of FGM plates to the CPT and FPT solutions are presented, which show that the present TPT solutions for the axisymmetric bending and buckling of the FGM circular plates are almost the same as the FPT solutions. Such that it can be concluded that the first-order shear deformation plate theory is enough to consider the effect of shear deformation on the axisymmetric bending and buckling of FGM plates. A much higher and more complex plate theory (say TPT) is not necessary in this case. By the way, it is seen that the present TPT solutions for the axisymmetric bending and buckling of FGM plates can provide useful benchmark to check the accuracy of related numerical results.

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